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Harmonic Analysis, Partial Differential Equations, Calculus of Variations, Geometric Measure Theory

A general principle in many areas of mathematics and science is that the dynamics of a system tend to evolve in a direction which reduces the available energy. This is the least action principle, which postulates the possibility of defining an energy - a functional whose input is taken from a set of possible states - and supposing that the output or “action” of that state prefers local minima. From this perspective, the calculus of variations is very important, since it is precisely the study of the extrema of functionals.

Our research focuses on the regularity of minimizers and how the qualitative properties depend on the differential order of the energy. Here we utilize an object whose study in the literature is surprisingly quite sparse – Riesz fractional gradients – and early research suggests that although there are significant differences in the properties of energies of integer differential order, there is no difference in the regularity or qualitative properties.

Thus far we have obtained several new existence and convergence results for the Partial Differential Equations we consider, as well as new inequalities that provide a nice complement to more classical results. Future research will look to new regularity results for non-linear problems as well as the application to physical problems modeled by these energies.

$$S = \int_{t_0}^{t_1} L(t, \mathbf{x}(t), \dot{\mathbf{x}}(t)) dt$$

$$\frac{\partial L(t, \mathbf{x}, \mathbf{v})}{\partial v_i} = mv_i = p_i$$

$$\frac{\partial L(t, \mathbf{x}, \mathbf{v})}{\partial x_i} = -\frac{\partial U(\mathbf{x})}{\partial x_i} = F_i(\mathbf{x})$$

$$L(t, \mathbf{x}, \mathbf{v}) = \frac{1}{2}m \sum_{i=1}^3 v_i^2 - U(\mathbf{x})$$