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Analytical, Geometrical, and Topological Properties of Viscous Incompressible Fluid Flows, Partial Differential Equations

The motion of an incompressible Newtonian fluid is described by the incompressible Navier-Stokes equation in the Euclidean space setting. Though the standard derivation of the incompressible Navier-Stokes equation based on the Newton's second law was well-understood already in the 19 century, serious mathematical study of viscous incompressible fluid flows from the view point of Partial Differential Equations only began with the existence theory of Leray-Hopf weak solutions to the incompressible Navier-Stokes equation as established by Leray (1930's) and Hopf (1950's). Since then, P.D.E. specialists in this area had devoted great efforts in obtaining the classical smoothness property of Leray-Hopf weak solutions under suitable space-time integrability conditions imposed on the weak solutions themselves. However, it is still a long standing open problem to decide whether the breakdown of classical smoothness of Leray-Hopf weak solutions could occur.

My research focuses on the study of analytical, geometrical, and topological properties of viscous fluid flows taking place in a curved space setting. In the past few years, I and professor Magdalena Czubak made a simple yet striking observation about the non-uniqueness phenomena of finite energy viscous fluid flows on a negatively curved surface of constant sectional curvature. and subsequently established the proper framework of weak solutions which restores uniqueness in the same setting. In the near future, we will try to find a deeper connection between mathematical properties of fluid flows taking place in a curved space setting and those of fluid flows taking place in the standard flat space setting.

$$\begin{aligned}\partial_t u - \Delta u + u \cdot \nabla u + \nabla P &= 0, \\ \operatorname{div} u &= 0.\end{aligned}\quad (0.1)$$

$$\partial_t \omega - \Delta \omega + [u, \omega] = 0. \quad (0.2)$$